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ANGULAR COEFFICIENTS IN SYSTEMS OF BODIES OF REVOLUTION

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UDC 536.33

The apparatus of differential geometry is used to calculate angular coefficients. Examples are given.

1. The angular coefficients between surfaces of revolution are widely used in calculating the radiant heat transfer in various metallurgical and power units: converters, vacuum units for degassing steel, furnaces for heating tubes and rolls of steel strip, recuperaters for heating air and gas, boiler units, etc.

The expression for the angular coefficient with an elementary area dS_M at an area dS_P (Fig. 1) in a diathermal medium takes the form

$$\varphi_{dM-dP} = \frac{\cos\alpha\cos\beta}{\pi|\mathbf{MP}|^2} dS_P. \tag{1}$$

The quantities cos α, cos β, and $|\mathrm{MP}|$ are found from the formulas [1]

$$\cos\alpha = \frac{(\mathbf{N}_M, \mathbf{MP})}{|\mathbf{N}_M|\,|\mathbf{MP}|}, \quad \cos\beta = -\frac{(\mathbf{N}_P, \mathbf{MP})}{|\mathbf{N}_P|\,|\mathbf{MP}|}, \tag{2}$$

$$|\mathbf{MP}| = (\mathbf{MP}, \mathbf{MP})^{1/2}, \tag{3}$$

where N_M, N_P are the vectors of the normals to the areas dS_M, dS_P.

Consider the case when dS_M and dS_P belong to surfaces of revolution S_M and S_P. Suppose that $S_M(S_P)$ is formed by revolution around the axis $Z_1(Z_2)$ of some curve in the plane $X_1O_1Z_1$ $(X_2O_2Z_2)$ of the rectangular Cartesian coordinate system $O_1X_1Y_1Z_1$ $(O_2X_2Y_2Z_2)$.

Introducing the spherical coordinates R, θ, φ, let $M = M(R(\theta), \theta, \varphi) = M(\theta, \varphi)$ be the radius vector of the point M. Then $N_M = M_\theta \times M_\varphi$, where M_θ, M_φ are vectors directed along the tangents to the coordinate lines φ = const and θ = const [2]. The projections of the vectors M, M_θ, M_φ, and N_M on the axes of the system $O_1X_1Y_1Z_1$ are given in Table 1. Knowing the projection N_M, it is simple to find $|N_M|$:

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TABLE 1. Projections of Vectors onto the Axes of a Rectangular Cartesian Coordinate System

Axes	$M = M(R, \theta, \varphi)$; R, θ, φ - are spherical coordinates; $R = R(\theta)$			
	vectors			
	M	$M_\theta = \partial M/\partial\theta$	$M_\varphi = \partial M/\partial\varphi$	$N_M = M_\theta \times M_\varphi$
X_1	$R\sin\theta\cos\varphi$	$(\dot{R}\sin\theta + R\cos\theta)\cos\varphi$	$-R\sin\theta\sin\varphi$	$(R\sin\theta - \dot{R}\cos\theta) R\sin\theta\cos\varphi$
Y_1	$R\sin\theta\sin\varphi$	$(\dot{R}\sin\theta + R\cos\theta)\sin\varphi$	$R\sin\theta\cos\varphi$	$(R\sin\theta - \dot{R}\cos\theta) R\sin\theta\sin\varphi$
Z_1	$R\cos\theta$	$\dot{R}\cos\theta - R\sin\theta$	0	$(R\cos\theta + \dot{R}\sin\theta) R\sin\theta$
Axes	$M = M(\rho, \psi, z)$; ρ, ψ, z are cylindrical coordinates; $\rho = \rho(z)$			
	vectors			
	M	$M_\psi = \partial M/\partial\psi$	$M_z = \partial M/\partial z$	$N_M = M_\psi \times M_z$
X_1	$\rho\cos\psi$	$-\rho\sin\psi$	$\dot{\rho}\cos\psi$	$\rho\cos\psi$
Y_1	$\rho\sin\psi$	$\rho\cos\psi$	$\dot{\rho}\sin\psi$	$\rho\sin\psi$
Z_1	z	0	1	$-\rho\dot{\rho}$

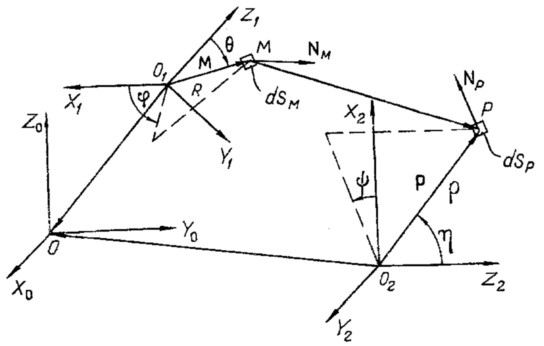

Fig. 1. Determining the angular coefficients between elementary areas.

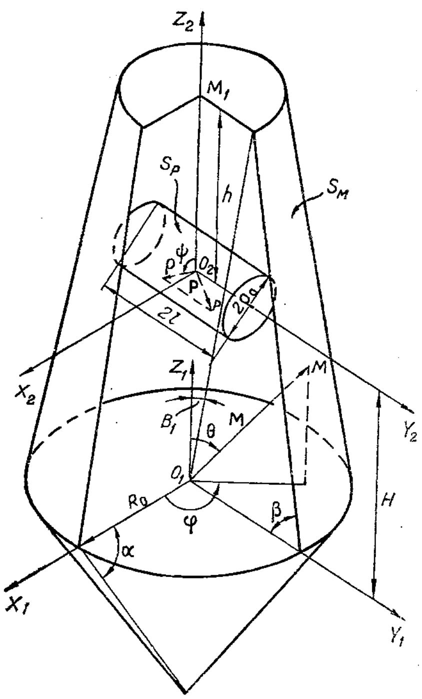

Fig. 2. Configuration of surfaces in examples (a) and (b).

$$|N_M| = (N_M, N_M)^{1/2} = R(\dot{R}^2 + R^2)^{1/2}\sin\theta.$$

The differential dS_M is determined as follows [2]:

$$dS_M = |N_M|\, d\theta d\varphi. \quad (4)$$

Analogous expressions are valid for $\mathbf{P}$, $\mathbf{P}_\eta$, $\mathbf{P}_\psi$, $\mathbf{N}_p$, and $|\mathbf{N}_p|$, where $\mathbf{P} = \mathbf{P}(\rho(\eta), \eta, \psi) = \mathbf{P}(\eta, \psi)$ is the radius vector of the point P (Fig. 1),

$$|\mathbf{N}_p| = \rho(\dot{\rho}^2 + \rho^2)^{1/2}\sin\eta, \quad dS_p = |\mathbf{N}_p|\, d\eta d\psi. \quad (5)$$

To calculate the scalar product of the vectors in Eqs. (2) and (3), consider the new rectangular Cartesian system $OX_0Y_0Z_0$ (it may coincide with one of the already existing systems); the projections of the vectors $\mathbf{MP}$, $\mathbf{N}_M$, $\mathbf{N}_p$ on its axes are now found.

Suppose that $\mathbf{O_1O}$ and $\mathbf{O_2O}$ are vectors connecting the origins of the old and new systems (Fig. 1), while T_1 and T_2 are matrices of coordinate transformation of the vector on passing to the new system [3]. It follows from geometric considerations that: $\mathbf{MP} = (\mathbf{P} - \mathbf{O_2O}) - (\mathbf{M} - \mathbf{O_1O})$. Hence

TABLE 2. Angular Coefficients ϕ_{dM-P} from the Element dS_M of the Surface S_M to the Surface S_P in Example (a) (Fig. 2)

Angle θ, deg	Angle φ, deg		
	0	45	90
0	0203	0203	0203
3	0195	0197	0199
18	0332	0389	1184
24	0232	0247	0278
30	0143	0132	0100
60	0031	0026	0021
86	0014	0012	0010
94	0046	0044	0043
120	0050	0050	0050
150	0049	0049	0049
180	0047	0047	0047

Note. The figures following the decimal point are given.

$$(\mathbf{MP})_0 = (\mathbf{P} - \mathbf{O}_2\mathbf{O})_0 - (\mathbf{M} - \mathbf{O}_1\mathbf{O})_0 = T_2^{-1}(\mathbf{P} - \mathbf{O}_2\mathbf{O})_2 - T_1^{-1}(\mathbf{M} - \mathbf{O}_1\mathbf{O})_1, \tag{6}$$

where $(a)_i$ is the column vector of the projections of a onto the axes X_i, Y_i, Z_i $(i = \overline{0, 2})$.

Analogously

$$(\mathbf{N}_M)_0 = T_1^{-1}(\mathbf{N}_M)_1,\ (\mathbf{N}_P)_0 = T_2^{-1}(\mathbf{N}_P)_2. \tag{7}$$

If S_P is the part of the surface of revolution corresponding to the region $G(\eta_1 \leq \eta \leq \eta_2, \psi_1 \leq \psi \leq \psi_2)$ of variation of the coordinates η and ψ, it follows from Eqs. (1) and (4) that

$$\varphi_{dM-P} = \int_{\eta_1}^{\eta_2}\int_{\psi_1}^{\psi_2} \frac{\cos\alpha\cos\beta}{\pi|\mathbf{MP}|^2}|\mathbf{N}_P|\,d\eta d\psi. \tag{8}$$

For S_M and S_P in the analogous case, using Eqs. (1), (4), and (5), it is found that

$$\varphi_{M-P} = \frac{1}{S_M}\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2}\int_{\eta_1}^{\eta_2}\int_{\psi_1}^{\psi_2} \frac{\cos\alpha\cos\beta}{\pi|\mathbf{MP}|^2}|\mathbf{N}_M|\,|\mathbf{N}_P|\,d\theta d\varphi d\eta d\psi, \tag{9}$$

where

$$S_M = \int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2}|\mathbf{N}_M|\,d\theta d\varphi.$$

2. The equations of the surfaces of revolution may be specified in any appropriate coordinate systems [4]. If, for example, the cylindrical system ρ, ψ, z is used, and the axis of revolution of the body coincides with the axis Z, the coordinates of the vectors **M** and $\mathbf{N}_M$ have [2] an entirely simple form (Table 1), while $|\mathbf{N}_M| = \rho(\rho^2 + 1)^{1/2}$. In Eqs. (8) and (9), only the variables of integration are changed in calculating the coefficients.

3. Consider some examples of calculating the angular coefficients by the given method.

a) Suppose that the surface S_M is formed by the internal surfaces of a cone and a truncated cone, while S_P is formed by the lateral surface of a cylinder (Fig. 2). It is required to find ϕ_{dM-P}.

The radius vector **M** of the point $M \in S_M$ in the spherical system has the coordinates R(θ), θ, φ:

$$R(\theta) = \begin{cases} (H+h)/\cos\theta, & 0 \leqslant \theta \leqslant \beta_1; \\ R_0\sin\beta/\cos(\beta-\theta), & \beta_1 < \theta \leqslant \pi/2; \\ -R_0\sin\alpha/\cos(\theta+\alpha), & \pi/2 < \theta \leqslant \pi. \end{cases} \tag{10}$$

The radius vector **P** of the point $P \in S_P$ in the cylindrical system has the coordinates ρ, ψ, y_2; $\rho(y_2) = \rho_0$.

TABLE 3. Results of Calculating φ_{dM_1-P} from Eq. (11) (upper row) and the Formula of [5] (lower row) for Various h/ρ_0 and ℓ/ρ_0

h/ρ_0	l/ρ_0			
	1	5	10	25
1,5	6272 (3,3)	6650 (3,8)	6685 (3,16)	6702 (3,16)
	6240	6661	6666	6666
2	3885 (3,3)	4969 (3,8)	4965 (3,8)	4988 (3,16)
	3894	4976	4997	4999
5	05876 (3,3)	1737 (3,3)	1984 (3,3)	1994 (3,8)
	05880	1740	1949	1996
10	01374 (3,3)	05842 (3,3)	08429 (3,3)	09818 (3,8)
	01374	05842	08438	09818
35	001063 (3,3)	005243 (3,3)	01007 (3,3)	02014 (3,3)
	001063	005243	01007	02014

Note. The figures after the decimal point are given; the number of points in the Gaussian quadrature when calculating the integral in Eq. (11) (taking account of the symmetry with respect to y_2) with respect to the coordinates ψ and y_2 is shown in parentheses.

The system $O_1X_1Y_1Z_1$ is obtained by parallel transfer of the system $O_2X_2Y_2Z_2$, while $(\mathbf{O}_2\mathbf{O}_1)_2' = (0, 0 - H)$. Suppose that the system $OX_0Y_0Z_0$ coincides with the system $O_1X_1Y_1Z_1$. Then T_1, T_2 are unit matrices; $\mathbf{O}_2\mathbf{O} = \mathbf{O}_2\mathbf{O}_1$, $\mathbf{O}_1\mathbf{O}$ is a zero vector; $(\mathbf{M})_1$ and $(\mathbf{N}_M)_1$ in Eqs. (6) and (7) are calculated from Table 1 using Eq. (10) for $R(\theta)$. In calculating $(\mathbf{P})_2$ and $(\mathbf{N}_p)_2$, it must be taken into account that the axis of revolution of the cylinder coincides with axis Y_2, while the polar angle is measured from the angle Z_2, so that in the first column of the lower part of Table 1 the axes must be placed in the order Z_1, X_1, Y_1; $(\mathbf{P} - \mathbf{O}_2\mathbf{O})_0' = (\rho_0 \sin\psi, y_2, \rho_0 \cos\psi)$, $(\mathbf{N}_P)_0' = (\rho_0 \sin\psi, 0, \rho_0 \cos\psi)$.

From a formula of the form of Eq. (8), taking account of the symmetry with respect to ψ, it is found that

$$\varphi_{dM-P} = \frac{2}{\pi} \int_{-l}^{l} dy_2 \int_{\psi_1}^{\psi_2} \frac{(\mathbf{N}_P, \mathbf{MP})(\mathbf{N}_M, \mathbf{MP})}{|\mathbf{MP}|^4 |\mathbf{N}_M|} d\psi, \tag{11}$$

where ℓ is half the cylinder length; $\psi_1 = \arccos(-(H-z_M)/u)$; $\psi_2 = \psi_1 + \arccos(\rho_0/u)$; $u = (x_M^2 + (H - z_M)^2)^{1/2}$, x_M, z_M are the projections of $\mathbf{M}$ on the axes X_1 and Z_1.

The Gauss formula is used to calculate the double integral on the right-hand side of Eq. (11). The calculations are performed by a program written in FORTRAN IV for an EC-1033 computer. Table 2 shows the values of ϕ_{dM-P} as a function of the angles θ and ϕ(ℓ/ρ_0 = 26; R_0/ρ_0 = 51.1; h/ρ_0 = 35.4; H/ρ_0 = 68.9; α = 13°; β = 70°).

According to the properties of mutuality and closure [5], the following relation holds

$$\varphi_{P-M} = 1 = \frac{1}{S_P} \int_{S_M} \varphi_{dM-P}\, dS_M. \tag{12}$$

Calculating the integral on the right-hand side of Eq. (12), it is found that φ_{P-M} = 0.985, i.e., the discrepancy with the accurate value is 1.5%.

b) The area dS_{M_1} is at the center of the upper base of the truncated cone. The normal to the area passes through the cylinder axis and perpendicular to it. For such a system, the formula expressing ϕ_{dM_1-P} in terms of elementary functions is known [5]. The results of the calculations from this formula and from Eq. (11) (taking account of the symmetry with respect to y_2) are in good agreement (Table 3).

c) A program has been written for calculating the angular coefficient between the lateral surfaces of nonintersecting finite cylinders that are arbitrarily positioned in space. Account is taken in the program that the area $dS_M \subset S_M$ may only influence the part of the cylinder S_P enclosed between two tangential planes to the cylinder S_P passing through the point M. Table 4 gives values of ϕ_{M-P} calculated by this program for cylinders of identical length whose axes form the angle α (Fig. 3). For parallel cylinders (α = 0), ϕ_{M-P} differs from the data of [6] by no more than 2%. The error in the results for cy linders with α = 45 and 90° is no greater than 5% according to present estimates.

TABLE 4. Angular Coefficients φ_{M-P} between Lateral Surfaces of Cylinders (Fig. 3). Upper Row: Present Calculation; Lower Row: [6]

R_0/ρ_0	l/ρ_0	s/ρ_0 = 0,5; α, deg: 0	45	90	s/ρ_0 = 2,0; α, deg: 0	45	90
0,1	0,5	0931	0065	0000	0151	0050	0000
		0933			0149*		
	1,0	1390	0232	0000	0287	0100	0000
		1387			0285*		
1,0	0,5	0514	0065	0000	0110	0041	0000
		0517			0110*		
10,0	0,5	0102	0027	00094	0032	0016	00045
		0104			0032		
	1,0	0167	0060	0017	0061	0031	00086
		0167			0062		
	5,0	0290	0145	0057	0187	0099	0036
		0288			0186		
	10,0	0308	0144	0058	0230	0112	0044
		0310*			0229		

Notes. 1. The figures following the decimal point are given. 2. An asterisk denotes values calculated by the approximate formula recommended in [6], where it was indicated that this formula approximates the accurate Eq. (3) of [6] with an error of less than 1%. However, the approximate formula includes an incorrect expression for φ^{∞}_{M-P} (φ_{M-P} when $\ell = \infty$) as a factor. The corrected expression used in the present calculations takes the form: $\varphi^{\infty}_{M-P} = (0.5/\pi)\,[((c/\rho_0)^2 - (R_0/\rho_0+1)^2)^{\frac{1}{2}} - ((c/\rho_0)^2 - (R_0/\rho_0-1)^2)^{1/2} + \pi + (R_0/\rho_0 - 1)\arccos((R_0-\rho_0)/c) - (R_0/\rho_0 + 1)\arccos((R_0 + \rho_0)/c)]$, where c is the distance between the axes of the cylinders.

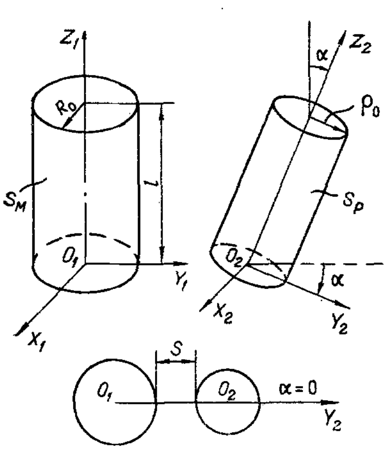

Fig. 3. Configuration of cylinders of finite length.

NOTATION

φ_{dM-dP}, φ_{dM-P}, angular coefficients from the elementary area dS_M to the elementary area dS_P and the surface S_P; φ_{M-P}, same from the surface S_M to surface S_P; **M**, **P**, radius vectors of points M and P; $\mathbf{N}_M$, $\mathbf{N}_P$, vectors normal to the areas dS_M and dS_P; T_i, matrices transforming the vector coordinates from the Cartesian system $O_iX_iY_iZ_i$ to the system $OX_0Y_0Z_0$ (i = 1, 2); $\mathbf{O_iO}$, vector connecting the origins of the systems $O_iX_iY_iZ_i$ and $OX_0Y_0Z_0$ (i = 1, 2).

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HEAT TRANSFER DURING CHEMICAL BOILING IN THE PRESENCE OF FREE CONVECTION

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Heat and mass transfer between a solid body and a liquid reagent in the presence of gas liberation is studied experimentally. The experimental results are generalized by a criterional dependence.

The term "chemical boiling" refers to the process of heterogeneous chemical interaction between a solid body and a liquid reagent, accompanied by the liberation of gas. Examples of such an interaction are reactions of metals with acids, as a result of which hydrogen is liberated in the form of bubbles. The bubbles forming on the surface of a solid body in the course of their growth and detachment make the boundary diffusion layer of the liquid turbulent, thereby intensifying mass transfer.

There is a qualitative and quantitative analogy between the processes under study and heat transfer accompanying boiling [1]. It is evident from the curves of the coefficient of mass transfer k versus the concentration of the reagent c_R, obtained for the interaction of magnesium with hydrochloric acid in the presence of free convection and presented in Fig. 1 (curves 1 and 2), that as the motive force (concentration) increases, the coefficient of mass transfer increases, reaches a maximum, and then decreases. The analog of concentration in a mass transfer process is the temperature difference in heat transfer, and the coefficient of mass transfer k is the analog of the coefficient of heat transfer α.

The analogy is confirmed experimentally for other characteristics of chemical boiling also. In particular, the rate of growth of the bubbles, the number of gas-formation centers, and the detachment diameters of the bubbles obey analogous laws of heat transfer accompanying boiling. For example, the investigation of the detachment diameter of CO_2 bubbles showed that its value is independent of the reagent concentration and is determined by the surface tension force (quasistatic regime). In the case of the detachment of H_2 bubbles the inertia from the side of the surrounding liquid plays the main role (dynamic regime), while the detachment diameter d_0, as in the case of heat transfer accompanying boiling, is proportional to $d_0 \sim Ja^{2/3}$.

The kinetic laws in the process of chemical boiling in the presence of forced convection are analogous. At low concentrations the transport of the reagent into the reaction zone plays the main role, while at high concentrations gas formation plays the main role [2].

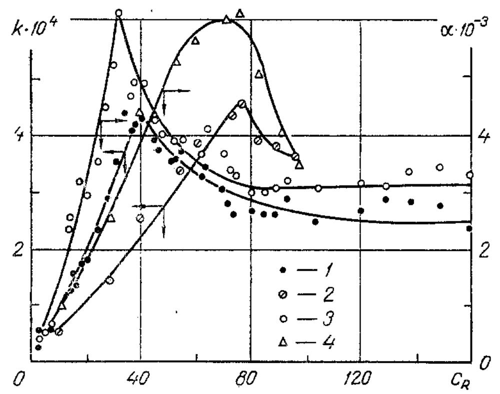

Fig. 1. Coefficients of mass transfer k (m/sec) and heat transfer α (W/(m·K)) versus the reagent concentration c_R (kg/m³) (1, 3: the initial temperature of the solution is equal to 40°C; 2, 4: the initial temperature of the solution is equal to 20°C).

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